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Bulk Viscus Fluid Plane Symmetric String Dust Magnetised Cosmological Model in General Relativity

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Abstract In this paper we have investigated bulk viscous fluid plane symmetric dust magnetized string cosmological model. To get a deterministic model, it is assumed that $\varepsilon = \lambda$, and a relation between metric potential $B = RA^n$. The behaviour of the model in the presence and absence of magnetic field together with physical and geometrical aspects of the model are also discussed.

Keywords Plane symmetric space-time · Bulk viscosity · Magnetized string cosmological model

1 Introduction

It is still problem to know the exact physical situation at the very early stages of the formation of large-scale structure of the universe. Cosmological models plays significant role in the evolution of early universe. These arise during the phase transition after the big bang explosion as the temp goes down below some critical temperature as predicted by grand unified theories [9, 19]. It is believed that cosmic strings may act as gravitational lenses and these objects are considered as possible seeds for formation of galaxies [20, 23]. The general relativistic treatment of string was initiated by Letelier [12, 13] and Stachel [16]. Letelier [12] has obtained the solution of Einstein's field equations for a cloud string with spherical, plane and cylindrical symmetry. Then, in 1983, he solved Einstein's field equations for cloud massive string and obtains cosmological models in Bianchi type-I and Kantowski-Sachs space-time. Benerjee et al. [5], have investigated an axially symmetric Bianchi type-I string dust

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cosmological model in presence and absence of magnetic field. Exact solutions of Bianchi type II, VI_o, VIII and IX space-times for a cloud string has been studied by Krori et al. [11]. The string cosmological models with magnetic field are also discussed by Tikekar and Patel [17] and Wang [21]. While Chakraborty and Chakraborty [8], Bhattacharya and Karade [7], Tikekar et al. [18] have presented the solutions of spherically symmetric models, axially symmetric models with dust source, and cylindrically symmetric models respectively in string cosmology.

It is interesting to note that magnetic field present, in galactic and intergalactic spaces play an important role in the cosmological scale. Melvin [15] in the cosmological solution for dust and electromagnetic field suggested that during the evolution of the universe, the matter was in highly ionized state and is smoothly coupled with the field and consequently from a neutral matter as a result of universe expansion. Hence in string dust universe, the presence of magnetic field is not unrealistic. Recently, Kilinc and Yavuz [10] studied string cosmological models with magnetic field in cylindrically symmetric space-time and Bali and Upadhaya [3], Bali and Anjali [4], Wang [22] have obtained Bianchi Type-I string cosmological models with viscosity in general relativity.

Motivated the situation discussed above, in this paper, we intend to construct a homogeneous plane symmetric bulk viscous fluid string dust magnetized cosmological models in general relativity. An equation $\varepsilon = \lambda$, and a relation between metric potential $B = RA^n$ are adopted.

2 Metric and Field Equations

We consider the plane symmetric metric

$$ds^2 = -dt^2 + A^2(dx^2 + dy^2) + B^2 dz^2, \tag{1}$$

where A and B are the functions of time only.

The energy momentum tensor for a cloud string with bulk viscosity [3] is,

$$T_i^j = \varepsilon v_i v^j - \lambda x_i x^j - \xi v^l_{i;l}(v_i v^j + g_i^j) + E_i^j, \tag{2}$$

where $\varepsilon = \varepsilon_p + \lambda$, is the rest energy density of the cloud string with particle attached to them, with ε_p is the rest energy density of particle and λ is the tension density of the cloud string, ξ is the coefficient of bulk viscosity. The vector v^i describes the flow velocity vector and x^i represents a direction of anisotropy, here we consider x^i be along z-axis. The direction of string satisfy the standard relations [2]

$$v^i v_i = -x^i x_i = -1 \quad \text{and} \quad v^i x_i = 0. \tag{3}$$

Here E_{ij} is the electromagnetic field given by [14]

$$E_{ij} = \overline{\mu}\left[|h|^2\left(v_i v_j + \frac{1}{2}g_{ij}\right) - h_i h_j\right], \tag{4}$$

where v_i is the flow vector satisfying

$$g_{ij} v^i v^j = -1. \tag{5}$$

$\overline{\mu}$ is the magnetic permeability and h_i the magnetic flux vector defined by

$$h_i = \frac{\sqrt{-g}}{2\overline{\mu}} \varepsilon_{ijkl} F^{kl} v^j, \tag{6}$$

where F^{kl} is the electromagnetic field tensor and ε_{ijkl} is the Levi-Civita tensor density.

In comoving co-ordinates, the field equations of Einstein in general relativity is

$$G_{ij} = -T_{ij}, \tag{7}$$

where G_{ij} is the Einstein tensor and we choose the units such that $C = 1, 8\pi G = 1$. The incident magnetic field is taken along z-axis so that,

$$h_1 = h_2 = h_4 = 0, \qquad h_3 \neq 0. \tag{8}$$

The first set of Maxwell's equation,

$$F_{ij;k} + F_{jk;i} + F_{ki;j} = 0 \tag{9}$$

leads to

$$F_{12} = \text{constant} = M(\text{say}). \tag{10}$$

Here $F_{14} = F_{24} = F_{34} = 0$, due to assumption of infinite electrical conductivity.

The only non-vanishing component of F_{ij} is F_{12}. Hence

$$h_3 = \frac{BM}{\overline{\mu} A^2}, \tag{11}$$

and

$$|h|^2 = h_l h^l = \frac{M^2}{\overline{\mu} A^4}. \tag{12}$$

From (4), we have

$$E_1^1 = \frac{M^2}{2\overline{\mu} A^2} = E_2^2 = -E_3^3 = -E_4^4. \tag{13}$$

With the help of (2) and (13), the field equations (7) for the metric (1) reduces to

$$\frac{A_{44}}{A} + \frac{A_4 B_4}{AB} + \frac{B_{44}}{B} = \xi v^l_{;l} - \frac{M^2}{2\overline{\mu} A^4}, \tag{14}$$

$$\left(\frac{A_4}{A}\right)^2 + 2\frac{A_{44}}{A} = \xi v^l_{;l} + \lambda + \frac{M^2}{2\overline{\mu} A^4}, \tag{15}$$

$$\left(\frac{A_4}{A}\right)^2 + 2\frac{A_4 B_4}{AB} = \varepsilon + \frac{M^2}{2\overline{\mu} A^4}. \tag{16}$$

Hereafter, the suffix 4 after A, B denotes ordinary differentiation with respect to time t.

3 Solution of the Field Equations

The set of field equations (14–16) are three equations containing five unknowns (A, B, λ, ε and ξ). In order to obtain a determinate solution, we assume that the rest energy density is equal to the string tension density, i.e.,

$$\varepsilon = \lambda, \tag{17}$$

and the relation between metric potentials

$$B = RA^n, \tag{18}$$

where R and n are both constant.

Eliminating λ, ε and ξ from (14–16) by using (17) and (18), we have

$$A_{44} + \frac{n(n+2)}{n-1}\frac{A_4^2}{A} = \frac{1}{1-n}\frac{M^2}{2\overline{\mu}A^3}. \tag{19}$$

For solving (19) we take $A_4 = \Upsilon$ then $A_{44} = \Upsilon\frac{d\Upsilon}{ds}$, which leads to

$$\Upsilon\frac{d\Upsilon}{dA} + \alpha\frac{\Upsilon^2}{A} = \frac{H}{(1-n)A^3}, \tag{20}$$

where

$$\alpha = \frac{n(n+2)}{n-1}, \tag{21}$$

$$H = \frac{M^2}{2\overline{\mu}}. \tag{22}$$

We obtained the solution of (20) as

$$\Upsilon = \left[\frac{-H}{n^2+n+1}A^{-2} + CA^{-2\alpha}\right]^{\frac{1}{2}}, \tag{23}$$

where C is the constant of integration.

Hence the line element (1) reduces to

$$ds^2 = -\left[\frac{-H}{n^2+n+1}A^{-2} + CA^{-2\alpha}\right]dA^2 + A^2(dx^2+dy^2) + R^2A^{2n}dz^2. \tag{24}$$

Under suitable co-ordinate transformations the metric (24) reduces to

$$ds^2 = -\left[\frac{-H}{n^2+n+1}T^{-2} + CT^{-2\alpha}\right]dT^2 + T^2(dX^2+dY^2) + R^2T^{2n}dZ^2. \tag{25}$$

4 Some Physical and Geometrical Properties

The energy density ε, the coefficient of bulk viscosity ξ, for model (25) are given by

$$\varepsilon = \lambda = (2n+1)CT^{\frac{-2(n^2+3n-1)}{n-1}} - \frac{n^2+3n+2}{n^2+n+1}HT^{-4}, \tag{26}$$

$$\xi = \left[\frac{2H}{(n+2)(n^2+n+1)}T^{-4} - \frac{2n}{n-1}CT^{\frac{-2(n^2+3n-1)}{n-1}}\right] \times \left[\frac{-H}{(n^2+n+1)}T^{-4} + CT^{\frac{-2(n^2+3n-1)}{n-1}}\right]^{-\frac{1}{2}} - \frac{2n}{(n+2)}\left[\frac{-H}{(n^2+n+1)}T^{-4} + CT^{\frac{-2(n^2+3n-1)}{n-1}}\right]^{\frac{1}{2}}. \tag{27}$$

The scalar of expansion θ, the shear scalar σ, and the spatial volume V are respectively given by

$$\theta = v^l_{;l} = (n+2)\left[\frac{-H}{(n^2+n+1)}T^{-4} + CT^{\frac{-2(n^2+3n-1)}{n-1}}\right]^{\frac{1}{2}}, \tag{28}$$

$$\sigma^2 = \frac{2}{3}(1-n)^2\left[\frac{-H}{(n^2+n+1)}T^{-4} + CT^{\frac{-2(n^2+3n-1)}{n-1}}\right], \tag{29}$$

$$V = RT^{n+2}. \tag{30}$$

5 Discussion

It is observed that from (26), the energy condition $\varepsilon \geq 0$ leads to

$$-\frac{(n^2+3n+2)H}{(2n+1)(n^2+n+1)C} \leq T^{\frac{-2(n^2+1)}{n-1}} \leq \frac{(n^2+3n+2)H}{(2n+1)(n^2+n+1)C}. \tag{31}$$

Therefore for the realistic model, we take span of time given by (31).

When $T \to 0$, then the energy density $\varepsilon \to \infty$ and the scalar of expansion $\theta \to \infty$; and when $T \to \infty$, then $\varepsilon \to 0$ and $\theta \to 0$ provided $\frac{(n^2+3n-1)}{n-1} > 0$. The cosmological model (25) represents shearing & nonrotating universe starts with a big bang.

Here it is observed that when $T \to 0$, then the spatial volume $V \to 0$ and when $T \to \infty$ then $V \to \infty$. These results show that the universe starts expanding with zero volume and blows up at infinite past and future.

Since $T \overset{\lim}{\to} \infty(\frac{\sigma}{\theta}) \neq 0$, the model (25) does not approach isotropy for large values of T in general which confirms that the universe remains anisotropic through the evolution except $n = 1$. When $n = 1$, then the shear scalar σ is zero. This shows that the model (25) isotropizes.

In the absence of magnetic field ($H = 0$), we obtain a string model with bulk viscosity,

$$ds^2 = -C^{-1}T^{\frac{2(n^2+2n)}{n-1}}dT^2 + T^2(dX^2+dY^2) + R^2T^{2n}dZ^2. \tag{32}$$

The energy density ε, the coefficient of bulk viscosity ξ, the scalar of expansion θ, the shear scalar σ, and the spatial volume V for model (32) are respectively given by

$$\varepsilon = \lambda = (2n+1)CT^{\frac{-2(n^2+3n-1)}{n-1}}, \tag{33}$$

$$\xi = -\frac{2n(2n+1)}{(n-1)(n+2)}C^{\frac{1}{2}}T^{\frac{-(n^2+3n-1)}{n-1}}, \tag{34}$$

$$\theta = (n+2)C^{\frac{1}{2}}T^{\frac{-(n^2+3n-1)}{n-1}}, \tag{35}$$

$$\sigma^2 = \frac{2}{3}(1-n)^2 C T^{\frac{-2(n^2+3n-1)}{n-1}}, \tag{36}$$

$$V = RT^{n+2}. \tag{37}$$

It is seen that from (33), the energy condition $\varepsilon \geq 0$ is satisfied when $n \geq -1/2$.

When $T \to 0$, then the energy density $\varepsilon \to \infty$ and the scalar of expansion $\theta \to \infty$; and when $T \to \infty$, then $\varepsilon \to 0$ and $\theta \to 0$ provided $\frac{(n^2+3n-1)}{n-1} > 0$. The cosmological model (32) represents shearing & nonrotating universe starts with a big bang.

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Since $T \overset{\lim}{\to} \infty(\frac{\sigma}{\theta}) \neq 0$, the model (32) does not approach isotropy for large values of T in general which confirms that the universe remains anisotropic through the evolution except $n = 1$. When $n = 1$, then the shear scalar σ is zero. This shows that the model (32) isotropizes.

From (33) and (34), we sees the relation between the coefficient of bulk viscosity and energy density i.e. $\xi \propto \varepsilon^{\frac{1}{2}}$ [1, 6].

Now, we obtain a string model without viscosity ($\xi = 0$) and magnetic field ($H = 0$),

$$ds^2 = -C^{-2}T^{\frac{2n^2}{n+1}}dT^2 + T^2(dX^2 + dY^2) + R^2T^{2n}dZ^2. \tag{38}$$

The physical and geometrical parameters are given by

$$\varepsilon = \lambda = (2n+1)C^2 T^{\frac{-2(n^2+n+1)}{n+1}}, \tag{39}$$

$$\theta = (n+2)CT^{\frac{-2(n^2+n+1)}{n+1}}, \tag{40}$$

$$\sigma^2 = \frac{2}{3}(1-n)^2 C^2 T^{\frac{-2(n^2+n+1)}{n+1}}, \tag{41}$$

$$V = RT^{n+2}. \tag{42}$$

Clearly, from (39), the energy condition $\varepsilon \geq 0$ is satisfied when $n \geq -1/2$.

When $T \to 0$, then the energy density $\varepsilon \to \infty$ and the scalar of expansion $\theta \to \infty$; and when $T \to \infty$, then $\varepsilon \to 0$ and $\theta \to 0$. The cosmological model (38) represents shearing & nonrotating universe starts with a big bang.

Here it is observed that when $T \to 0$, then the spatial volume $V \to 0$ and when $T \to \infty$ then $V \to \infty$.

Since $T \overset{\lim}{\to} \infty(\frac{\sigma}{\theta}) \neq 0$, the model (32) does not approach isotropy for large values of T. When $n = 1$, then the shear scalar σ is zero. This shows that the model (32) isotropizes.

6 Conclusion

We have investigated anisotropic homogeneous plane symmetric cosmological model with bulk viscous fluid string dust magnetized cosmological model. Generally the models are expanding, shearing and nonrotating. In all these models, we observe that they do not approaches isotropy for large values of T. Here we assumed that $\varepsilon = \lambda$, and a relation between metric potential $B = RA^n$. In absence of magnetic field, the solution reduces to string model with bulk viscosity. Also, we obtain string model without bulk viscosity and magnetic field. The physical and geometrical aspects of the model are also discussed.

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